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SUMMARY

An analysis of pressure and flow perturbations produced in a propellant feed system by fundamental-mode longitudinal oscillations of the structure of the vehicle is used to derive an expression for transfer function that relates the pump-flow perturbation to unit structural oscillation velocity for a system in which a compensating-bellows device is located between the feed line and the pump inlet. The device consists essentially of an enclosed variable volume that is controlled by the relative motion of two parts of the structure that oscillate with different amplitudes. Results of the analysis indicate that pump-inlet flow perturbations can be suppressed by a suitable selection of the compensating-bellows parameters.

INTRODUCTION

Longitudinal structural oscillations (0 to 30 cps) occurring in launch vehicles near the end of the boost phase have become a problem of increasing concern with the trend to heavier space-vehicle payloads. As larger payload space missions involving heavily loaded multistage vehicles are planned, increasing attention will have to be given to the longitudinal stability characteristics of the boost vehicle.

The mechanism of longitudinal instability apparently involves propellant-system flow perturbations driven by structural oscillations, which in turn produce oscillations in engine thrust. The thrust perturbations are fed back through the structure to complete the mechanism. Although all aspects of the problem are not fully understood, various dynamic models have been proposed to study this problem (ref. 1). Reference 1 presents one of the more complete models of the system, which includes, for example, a distributed-parameter propellent line, and the effects of mechanical oscillations of various parts of the propellant system. A similar model is described in an unpublished report by Dr. S. Rubin of Aerospace Corporation. These models provide a means of including the effects of proposed oscillation-suppression devices. The device discussed herein represents a generalization of the concept embodied in the constant-volume bellows proposed by Douglas Aircraft Company for use on the fuel side of the vehicle.

The very complexity of the complete model for the oscillating system tends to obscure the fundamental principles that must be considered in the design of flow-suppression devices. For this reason a simplified analytical propellant-system model was developed at the NASA Lewis Research Center. This report presents a simplified analysis of the feed system for one propellant when a compensating bellows device is used for suppression of flow perturbations at the inlet of the pump. By using the methods presented in references 2 to 4, the transfer function expressing pump-inlet flow perturbation as a response to unit structural velocity perturbation is obtained. This transfer function provides a means of selecting an optimum value for the compensating bellows parameter.

ANALYSIS

Since longitudinal oscillations have been a problem only near the end of the booster-burning period, a condition of nearly empty tanks is assumed. The vehicle structure, including engines, payload, and a small amount of propellants in the tanks, is assumed to be oscillating sinusoidally in the fundamental longitudinal mode at or very close to the natural frequency. Under these conditions a nodal point exists near the upper end of the booster. The engine and pump assembly is treated as a rigid unit with an oscillation velocity $\dot{x}_1$. (Symbols are defined in the appendix.) The velocity of oscillation of the bottom of the propellant tank $\dot{x}_t$ is related to $\dot{x}_1$ by the mode-shape factor φ_t as

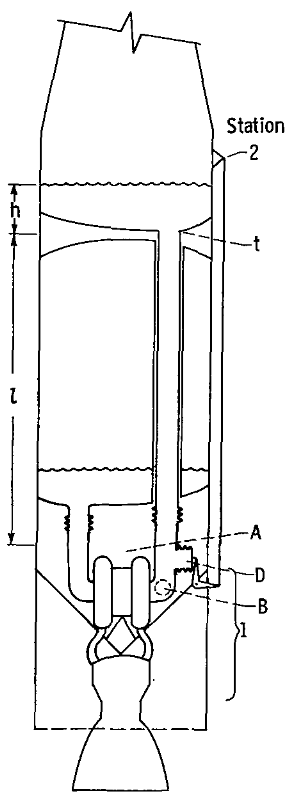


Figure 1. - Schematic representation of propellant feed system.

$$\dot{x}_t = \varphi_t \dot{x}_1 \tag{1}$$

Since the structure is lightly damped, the imaginary part of φ is small compared with 1 and small compared with the real part except near a nodal point. (Complex numbers are used to represent sinusoidal oscillations according to the usual conventions.)

Pressure and volume-flow perturbations in the propellant feed line are to be described by acoustic equations. This description requires that the flow perturbation be measured in a reference system that does not oscillate. Accordingly, reference station A (fig. 1) was selected to represent the mean position of the lower end of the feed line. Station A moves with the vehicle but does not oscillate. The compressibility in the pump-inlet region associated with cavitation in the inducer stage is represented symbolically by the bubble at the pump inlet (station B, fig. 1). The compressibility is represented analytically by C_B, the decrease in total volume of vapor bubbles in the pump-inlet region per unit increase in suction pressure under operating conditions. The flow into the bubble in response to a sinusoidal variation of suction pressure is given by

$$Q_B = \Delta P_s (i\omega C_B) \tag{2}$$

The volume compensating device, shown at station D in figure 1, is attached to the feed line at the inlet of the pump. It consists essentially of a variable volume controlled by the relative displacement of two parts of the structure that oscillate with different amplitudes. The compensator is attached to the pump assembly and therefore has the oscillation displacement x_1. It is connected by means of a mechanical linkage (or servomechanism) to a point in the forward part of the structure having a displacement $x_2 = \varphi_2 x_1$. The arrangement shown in figure 1 illustrates a kinematic principle of operation but is not intended to represent practical hardware. The volume enclosed by the compensator is related to the displacement difference $x_1 - x_2$ by the following equation that defines the kinematic factor k:

$$V = V_o + kS(x_1 - x_2) \tag{3}$$

The volume flow into the compensator is

$$Q_D = \dot{V} = kS\dot{x}_1(1 - \varphi_2) \tag{4}$$

The perturbation of flow through the pump is given by the following flow-continuity equation:

$$\Delta Q_p = \Delta Q_A + S\dot{x}_1 - Q_B - Q_D \tag{5}$$

where ΔQ_A (positive downward) is the flow perturbation at the lower end of the feed line with respect to the nonoscillating reference station A, and $S\dot{x}_1$ (positive upward) is the flow equivalent of the oscillation velocity of the pump assembly.

The flow perturbation ΔQ_A depends on the pressure perturbations at both ends of the propellant feed line. The acoustic transmission line equation (ref. 3) relating ΔP_t to ΔQ_A and ΔP_s is

$$\Delta P_t = \Delta P_s \cos \beta + iZ_o \, \Delta Q_A \sin \beta \tag{6}$$

where $\beta = \omega l/a$ and $Z_o = \rho a/S$.

The angle β is the phase shift in the line length l for a single sinusoidal wave train with propagation speed a, and Z_o is the characteristic acoustic impedance of the line. Since the distance from station A to the pump inlet is negligible compared with l, the suction-pressure perturbation ΔP_s is used in equation (6) for the perturbation at station A. For booster vehicles now in use the value of β is about 0.1 radian for the short line and 1.0 to 1.5 radians for the long line. Equation (6), therefore, can be solved for ΔQ_A

$$\Delta Q_A = \frac{\Delta P_t}{iZ_o \sin \beta} - \frac{\Delta P_s}{iZ_o \tan \beta} \tag{7}$$

with the reservation that the following analysis is not valid when β approaches zero or π.

Acoustic effects in the propellant tank can be neglected in determining the perturbation pressure ΔP_t at the tank outlet because the height h of propellant in the tank is small compared with 1/4 wavelength. Justifiably, the effect on ΔP_t of the tank-outlet flow perturbation can also be neglected because the area of the feed line is only about 1 percent of the area of the tank. With these approximations,

$$\Delta P_t = \rho h \ddot{x}_t \tag{8}$$

Replacing $\ddot{x}_t$ by $i\omega\varphi_t\dot{x}_1$ and using the parameters β and Z_o give

$$\Delta P_t = iS\dot{x}_1 Z_o \beta \frac{h}{l} \varphi_t \tag{9}$$

Substituting ΔP_t from equation (9) into equation (7) gives

$$\Delta Q_A = S\dot{x}_1 \frac{\beta}{\sin\beta} \frac{h}{l} \varphi_t - \frac{\Delta P_s}{iZ_o \tan\beta} \tag{10}$$

Substituting of equations (2), (4), and (10) into equation (5) gives the pump-flow perturbation as a function of the structural oscillation velocity and the suction pressure perturbation:

$$\Delta Q_p = S\dot{x}_1 \left[1 + \frac{\beta}{\sin\beta} \frac{h}{l} \varphi_t - k(1 - \varphi_2)\right] - i\,\Delta P_s \left(\omega C_B - \frac{1}{Z_o \tan\beta}\right) \tag{11}$$

The apparent impedance, looking into the pump from the suction side, is defined by

$$Z_{p,s} = \frac{\Delta P_s}{\Delta Q_p} \tag{12}$$

The transfer function defining the pump-flow response to structural oscillation is obtained from equation (11) by using $Z_{p,s}$ formally to eliminate ΔP_s

$$\frac{\Delta Q_p}{S\dot{x}_1} = \frac{1 + \frac{\beta}{\sin\beta} \frac{h}{l} \varphi_t - k(1 - \varphi_2)}{1 + iZ_{p,s}\left(\omega C_B - \frac{1}{Z_o \tan\beta}\right)} \tag{13}$$

The quantity $Z_{p,s}$ depends on pump characteristics, mean suction pressure, and other factors including the amplitude and phase of the thrust-chamber perturbation pressure. Use of the term impedance is not intended to imply that $Z_{p,s}$ has the properties usually associated with the impedance of a passive element or network. Very little information is available concerning either C_B or

$Z_{p,s}$, but it is probably safe to assume that the denominator in equation (13) is not zero.

The pump-flow response to unit structural oscillation velocity can be minimized by adjusting the factor k to make the numerator in equation (13) pass through zero at the time in flight when the likelihood of oscillation is greatest. Thus, the optimum value of k is

$$k_{opt} = \frac{1 + \frac{\beta}{\sin \beta} \frac{h}{l} \varphi_t}{1 - \varphi_2} \tag{14}$$

where the values of φ_t, φ_2, h, and β are taken at 5 to 10 seconds before booster engine cutoff.

Examination of equation (13) suggests another possible method of reducing the pump-flow perturbation. If the pump-inlet compliance could be greatly increased, and thus $\omega C_B - (1/Z_o \tan \beta)$ could be made large compared with $1/Z_{p,s}$, the denominator in equation (13) would be made large compared with 1, and the ratio $\Delta Q_p/S\dot{x}_1$ would thereby be reduced. The required increase in compliance could be realized by attaching next to the pump inlet an accumulator having a large compliance and small inertia and resistance. This arrangement would have the advantage of introducing a phase lag approaching 90° between ΔQ_p and $\dot{x}_1$.

Physical Interpretation

An examination of equation (11) provides a means of understanding the physical mechanism linking structural oscillations and propellant-flow perturbations. This flow-continuity equation can be rewritten as follows with motion-sensitive terms on the left side and pressure-sensitive terms on the right:

$$S\dot{x}_1 + \frac{h}{l} \frac{\beta}{\sin \beta} \varphi_t S\dot{x}_1 - k(1 - \varphi_2)S\dot{x}_1 = \Delta Q_p + i\omega C_B \Delta P_s + \frac{\Delta P_s}{iZ_o \tan \beta} \tag{11a}$$

The left side represents a net flow-perturbation input at the lower end of the propellant line resulting from mechanical oscillation of the structure. The first term on the left side is the direct pumping action of the oscillation velocity $\dot{x}_1$ of the pump assembly; the second term is the effect of the tank-pressure perturbation caused by the oscillation velocity $\varphi_t\dot{x}_1$ of the tank bottom as modified by the acoustic transmission characteristics of the feed line; and the third term is the compensating effect of the variable-volume device. This input flow perturbation goes out in the three directions. One component (ΔQ_p) goes through the pump, another ($i \Delta P_s \omega C_B$) is absorbed by the pump-inlet compliance, and the third ($\Delta P_s/iZ_o \tan \beta$) goes back into the feed line. The total perturbation flow is divided among the three channels according to their respective admittances, $1/Z_{p,s}$, $i\omega C_B$, and $1/iZ_o \tan \beta$. The phasor sum of the three outflow components equals the input flow perturbation. The pump suction-pressure perturbation is determined by the input flow perturbation and the resultant impedance of the three parallel channels; thus,

$$\Delta P_s = S\dot{x}_1\left[1 + \frac{\beta}{\sin\beta}\frac{h}{l}\varphi_t - k(1 - \varphi_2)\right]\frac{1}{\frac{1}{Z_{p,s}} + i\omega C_B + \frac{1}{iZ_o \tan\beta}} \tag{15}$$

The apparent pump-inlet impedance $Z_{p,s}$ defined in equation (12) depends on the entire system downstream of the pump inlet and the relation between pump flow and thrust-chamber pressure, but chamber pressure is also a function of flow perturbations in the other propellant system. A mechanism of interaction between the fuel and oxidant systems exists, therefore, and a reduction in flow oscillations in one system may possibly lead to increased oscillation in the other.

Application

The volume compensator consists essentially of a variable volume controlled by the relative displacement of two parts of the structure that oscillate with different amplitudes. The device is located next to the pump inlet and requires for its operation a mechanical connection to some point far enough forward on the vehicle to have an oscillation amplitude significantly smaller than that at the engine section.

On space-launch vehicles in which the long propellant line is firmly attached to the bottom of the upper tank, the motion of the lower end of the feed line may be used to actuate a compensating bellows device on the long-line system. (The mode factors φ_t and φ_2 would be the same in this case.) By means of a suitable linkage, the relative motion of the lower end of the long propellant line might also be used to drive a volume compensator on the short-line side.

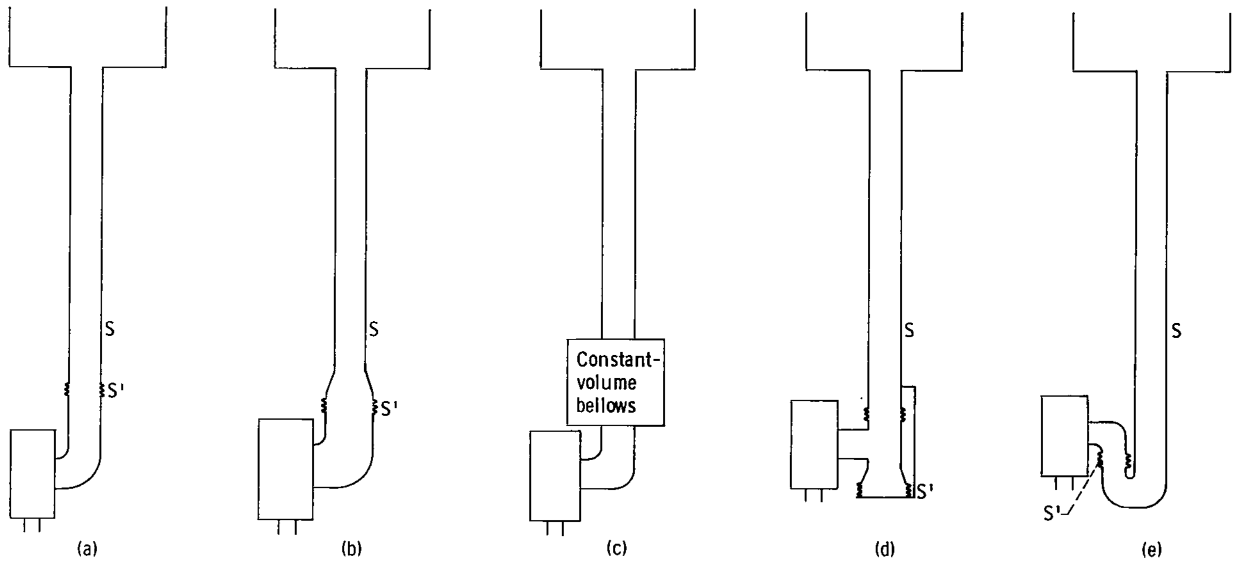
On vehicles in which a bellows is located between the upper tank and the long propellant line no convenient source of forward-structure motion is available in the engine section. For such vehicles a relative-motion signal could be obtained from a taut wire or ribbon connected to some forward point on the vehicle structure, but a servomechanism would probably be required to provide the necessary stiffness.

The kinematic factor k depends on the dimensions of the device, the geometry of the propellant system, and the kinematics of the actuating linkage. The determination of k for a particular configuration is accomplished by comparing it with the configuration of figure 1 (p. 2) (used to define k) in terms of the propellant-system volume changes caused by unit upward displacements of the pump assembly and the upper connection point. The comparison is facilitated by using a form of the flow equation in which the effects of pump motion $\dot{x}_1$ and upper-structure motion $\varphi_2\dot{x}_1$ are represented by separate terms. Substituting from equation (4) into equation (5) and rearranging terms yield

$$\Delta Q_p - Q_B - \Delta Q_A = (S - kS)\dot{x}_1 + (kS)\varphi_2\dot{x}_1 \tag{16}$$

From figure 1 it can be seen that $S - kS$, the coefficient of $\dot{x}_1$, represents the decrease in volume below station A that would result if the pump assembly were given a unit upward displacement without moving the upper connection point (station 2). The coefficient of $\varphi_2\dot{x}_1$ is related in a similar way to motion of the upper point. Thus, a unit upward displacement of point 2 with the pump fixed causes the volume below station A to decrease by kS.

Certain propellant-system configurations, in which the motion of the bottom of the upper tank is transmitted to the engine section by the propellant line, are shown in figure 2. An equation analogous to equation (16) can be written



(a) k = 0; straight bellows; bellows area S' equal to propellant line area S.
(b) k = (S - S')/S; straight bellows; bellows area S' not equal to propellant line area S.
(c) k = 1; constant-volume bellows.
(d) k = S'/S; compensating bellows with T-shaped pump-inlet configuration.
(e) k = 1 + S'/S; inverted bellows with reverse bend in propellant line.

Figure 2. - Values of kinematic factor k for various possible configurations in which propellant line moves with tank. Forward-structure and tank-bottom mode shape factors equal.

for each of these configurations by using the system geometry to determine the coefficients of $\dot{x}_1$ and $\varphi_t\dot{x}_1$. Thus, for figure 2(b), the volume decrease caused by unit upward displacement of the pump is S', and the volume decrease caused by unit upward displacement of the tank and feed line is -(S' - S). The flow equation is, therefore,

$$\Delta Q_p + Q_B - \Delta Q_A = (S')\dot{x}_1 + (S - S')\varphi_t\dot{x}_1 \qquad (17)$$

Equations (16) and (17) are identical if $k = (S - S')/S$ and $\varphi_2 = \varphi_t$. This procedure gives the following results for the configurations shown in figure 2 (at the left).

Configuration in figure -	Expression for $\Delta Q_p + Q_B - \Delta Q_A$	Expression for k
2(a)	$(S)\dot{x}_1 + (0)\varphi_t\dot{x}_1$	0
2(b)	$(S')\dot{x}_1 + (S - S')\varphi_t\dot{x}_1$	(S - S')/S
2(c)	$(0)\dot{x}_1 + (S)\varphi_t\dot{x}_1$	1
2(d)	$(S - S')\dot{x}_1 + (S')\varphi_t\dot{x}_1$	S'/S
2(e)	$(-S')\dot{x}_1 + (S' + S)\varphi_t\dot{x}_1$	(S + S')/S

CONCLUDING REMARKS

The results of this analysis suggest that the use of a volume compensator on either the fuel or the oxidant system would effectively minimize propellant-flow response to structural oscillations for that system, but because of possible interactions, would not necessarily eliminate oscillation of the vehicle.

Lewis Research Center
National Aeronautics and Space Administration
Cleveland, Ohio, June 1, 1964

APPENDIX - SYMBOLS

a effective sonic speed in propellant in feed line, ft/sec

C compliance, cu ft/(lb/sq ft)

h liquid level in propellant tank, ft

i imaginary number (complex numbers are used to represent sinusoidal oscillations according to usual convention)

k kinematic factor (characterizes volume compensator), dimensionless

l length of propellant feed line, ft

ΔP perturbation pressure, lb/sq ft

Q volume flow rate, cu ft/sec

ΔQ perturbation flow rate, cu ft/sec

S area of cross section of feed line, sq ft

S' area of cross section of bellows, sq ft

V volume of compensator, cu ft

V_o equilibrium volume of compensator, cu ft

x linear displacement with respect to nonoscillating reference system moving with vehicle, ft

Z acoustic impedance, (lb/sq ft)/(cu ft/sec)

Z_o characteristic acoustic impedance

β sonic delay-time phase shift, radians

ρ propellant density, slugs/cu ft

φ mode-shape factor, dimensionless

ω first-mode natural frequency, radians/sec

Subscripts:

A nonoscillating reference station at lower end of line

B pump-inlet

D compensator

p	through pump
s	pump suction side
t	bottom of propellant tank
1	engine section
2	forward structure connection point

Superscripts:

$(\dot{\ })$, $(\ddot{\ })$	first and second derivatives with respect to time

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